FIN 538

# Problem Set 2 – Solutions

①

a. $Pr(X \leq 0) = \phi\left(\frac{0-12}{20}\right) = \phi(-0.60) = 0.2743$

b. $Pr(X \leq -4) = \phi\left(\frac{-4-12}{20}\right) = \phi(-0.80) = 0.2119$

c. $Pr(X > 8) = 1 - Pr(X \leq 8)$

$= 1 - \phi\left(\frac{8-12}{20}\right) = 1 - \phi(-0.20)$

$= 0.5793$

d. $Pr(4 < X \leq 10) = Pr(X \leq 10) - Pr(X \leq 4)$

$= \phi\left(\frac{10-12}{20}\right) - \phi\left(\frac{4-12}{20}\right)$

$= \phi(-0.10) - \phi(-0.40)$

$= 0.1156$

② a. $z_{0.05} = 1.64 \Rightarrow$ 90% CI for X $[-22.90, 42.90]$

b. $z_{0.025} = 1.96 \Rightarrow$ 95% CI for X $[-29.20, 49.20]$

c. $z_{0.005} = 2.58 \Rightarrow$ 99% CI for X $[-41.52, 61.52]$

③ a. $\Pr(Y \leq 6) = \Pr\left(Z \leq \frac{\ln 6 - 3.9}{15}\right)$

$= \phi(-0.1415)$

$= 0.4441$

b. $\Pr(Y > 4) = 1 - \Pr(Y \leq 4)$

$= 1 - \Pr\left(Z \leq \frac{\ln 4 - 3.9}{15}\right)$

$= 1 - \phi(-0.1676)$

$= 0.5665$

c. $\Pr(3 < Y \leq 12) = P(Y \leq 12) - P(Y \leq 3)$

$= \Pr Z\left(\frac{\ln 12 - 3.9}{15}\right) - \Pr\left(Z \leq \frac{\ln 3 - 3.9}{15}\right)$

$= \phi(-0.0943) - \phi(-0.1868)$

$= 0.4624 - 0.4259$

$= 0.3665$

d. $P(Y \leq 0) = 0.$

④ $P(Y > 45) = 1 - P(Y \leq 45)$

$$= 1 - P\left(Z \leq \frac{\ln 45 - 3.70}{0.80}\right)$$

$$= 1 - \phi(0.1333)$$

$$= 1 - 0.5530$$

$$= 0.4470$$

⑤ a. $EY = \exp(1 + \frac{1}{2} 2^2) = 20.09$

b. $EY^2 = \exp(2 \cdot 1 + \frac{1}{2} 2^2 2^2) = 22026.47$

$(EY)^2 = 403.43$

$SD(Y) = \sqrt{22026.47 - 403.43} = 147.05$

c. $E(Y^{0.3}) = \exp(0.3 \cdot 1 + \frac{1}{2} 0.3^2 2^2) = 1.62$

d. $EY^{-1} = \exp(-1 \cdot 1 + \frac{1}{2}(-1)^2 2^2) = 2.72.$

⑥ a. $E\, S_T = 100 \exp(0.12 \cdot 2) = 127.12$

b. $E \ln S_T = \ln(100) + (0.12 - \tfrac{1}{2}\, 0.35^2)2 = 4.7227$

$\sigma \ln S_T = 0.35\sqrt{2} = 0.4950.$

c. $$P(S_T \leq 100) = P\left(Z \leq \frac{\ln 100 - 4.7227}{0.4950}\right)$$

$$= \phi(-0.24)$$

$$= 0.4052.$$

d. $$P(S_T > 120) = 1 - P(S_T \leq 120)$$

$$= 1 - P\left(Z \leq \frac{\ln(120) - 4.7227}{0.4950}\right)$$

$$= 1 - \phi(0.13)$$

$$= 0.4482$$

(7)

a. $E S_T = 60 \exp(0.18 \times 9/12) = 68.67$

b. $E \ln S_T = \ln 60 + (0.18 - \frac{1}{2}\, 0.32^2)\, 9/12 = 4.1909$

$\sigma \ln S_T = 0.32 \sqrt{9/12} = 0.2771$

c. The 95% CI for $\ln S_T$ is

$$[4.1902 - 1.96 \times 0.2771,\ 4.1902 + 1.96 \times 0.2771]$$

$$= [3.6478,\ 4.7341]$$

Thus, the corresponding CI for $S_T$ is

$$[38.39,\ 113.76]$$